Solve the following Differential Equations:

1) $\left(x + \tan^{-1} y\right)dx + \left(\dfrac{x+y}{1+y^2}\right)dy = 0$

$$M = x + \tan^{-1}(y) \implies \frac{\partial M}{\partial y} = \frac{1}{1+y^2}$$

$$N = \frac{x+y}{1+y^2} \implies \frac{\partial N}{\partial x} = \frac{1}{1+y^2}$$

same so we can find solution

$$F(x,y) = \int M\,dx = \int x + \tan^{-1}y \, dx = \tfrac{1}{2}x^2 + (\tan^{-1}y)x + g(y)$$

$$\frac{x+y}{1+y^2} = N = \frac{\partial F}{\partial y} = \frac{x}{1+y^2} + g'(y)$$

$$\implies \frac{y}{1+y^2} = g'(y)$$

$$\implies g(y) = \int \frac{y}{1+y^2}\,dy = \tfrac{1}{2}\ln(1+y^2) + c$$

$\therefore$ Answer is $\quad \tfrac{1}{2}x^2 + x\tan^{-1}y + \tfrac{1}{2}\ln(1+y^2) = c$

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2) $\left(\dfrac{2x^{5/2} - 3y^{5/3}}{2x^{5/2}y^{5/3}}\right)dx + \left(\dfrac{3y^{5/3} - 2x^{5/2}}{3y^{5/3}x^{5/2}}\right)dy = 0$

$$M = \frac{2x^{5/2} - 3y^{5/3}}{2x^{5/2}y^{5/3}} \implies \frac{\partial M}{\partial y} = \frac{\partial}{\partial y}\left(y^{-5/3} - \frac{3}{2}x^{-5/2}\right) = -\frac{5}{3}y^{-8/3}$$

$$N = \frac{3y^{5/3} - 2x^{5/2}}{3y^{5/3}x^{5/2}} = x^{-5/2} - \frac{2}{3}y^{-5/3} \implies \frac{\partial N}{\partial x} = -\frac{5}{2}x^{-7/2}$$

since $\dfrac{\partial M}{\partial y} \neq \dfrac{\partial N}{\partial x}$

we can't find $F(x,y)$ such that

$F(x,y) = c$ is a solution

to our D.E.